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COMMENT

Cylindrical shells of cosmic strings

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Abstract. The spacetime metric corresponding to a cylindrical shell made of straight line cosmic strings is presented.

The spacetime structure due to cosmic strings of infinite length has been studied extensively in recent years in terms of solid cylinder models of finite, as well as of vanishing, cross section (line sources) [1]. In both cases, the characteristic angular defect that obtains turns out to be equal to $8\pi\mu$, where μ is the linear mass density of the source, when the radial stress is neglected [2]. The metric corresponding to N parallel straight line cosmic strings of the above type has also been obtained [3].

In the present comment we consider the case of a continuous distribution of cosmic strings in the form of a static right cylindrical shell. We find that, in the shell's exterior, the spacetime manifold obtains a conical structure with an angular defect which is equal to the one arising in the presence of a solid cylinder of cosmic strings having the same linear mass density. This reinforces the analogy noted by Xanthopoulos [4] between electric currents and magnetic fields on the one hand, and cosmic strings and global gravitational effects, on the other because, in terms of this analogue, the present solution corresponds to a surface current parallel to the cylinder's axis.

Let us assume, then, that in the coordinate system (t, ρ, φ, z) with $t, z \in (-\infty, \infty)$, $\rho \in (0, \infty)$ and $\varphi \in (0, 2n)$ the line elements

$$ds_-^2 = -dt^2 + d\rho^2 + \rho^2\, d\varphi^2 + dz^2 \tag{1}$$

and

$$ds_+^2 = -dt^2 + d\rho^2 + \rho_0^2\left(\frac{\rho + b}{\rho_0 + b}\right)^2 d\varphi^2 + dz^2 \tag{2}$$

express the metric properties of spacetime in the shell's interior and exterior, respectively. In (2) ρ_0 is the sectional radius of the shell and b is a non-negative constant.

We will show that, by choosing b appropriately, (1) and (2) provide an exact solution of Einstein's equations with the shell as a source. In order to do this it suffices to prove that the following conditions are satisfied [5].

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(i) The metric coefficients g^-_{ab} and g^+_{ab}, where a, $b=0$, 1, 2, 3 corresponding to t, ρ, φ and z, satisfy Einstein's vacuum equations.

(ii) $g^-_{ab}(\rho_0)=g^+_{ab}(\rho_0)$.

(iii) $$\gamma^i_j-\delta^i_j\,\mathrm{Tr}(\gamma)=8\pi S^i_j \tag{3}$$

where

$$\gamma_{ij}\equiv K^+_{ij}(\rho_0)-K^-_{ij}(\rho_0) \tag{4}$$

$$K_{ij}\equiv-\tfrac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\rho}(g_{ij}). \tag{5}$$

Here, $i, j=0, 2, 3$, K_{ij} is the 'extrinsic curvature tensor' of a $\rho=$ constant hypersurface and S_{ij} is the 'surface stress-energy tensor' of the shell.

Now, the linear coordinate transformation

$$\bar{\rho}=\rho+b \qquad \bar{\varphi}=\frac{\rho_0}{\rho_0+b}\varphi \tag{6}$$

turns the line element (2) into

$$\mathrm{d}s^2_+=-\mathrm{d}t^2+\mathrm{d}\bar{\rho}^2+\bar{\rho}^2\,\mathrm{d}\bar{\varphi}^2+\mathrm{d}z^2 \tag{7}$$

which is identical in form to (1). The latter is no more than the line element of Minkowski space in cylindrical coordinates and this implies that condition (i) above is satisfied trivially.

From (1) and (2) it is obvious that condition (ii) is also satisfied. Using the same equations, on the other hand, in combination with definitions (4) and (5), we find that the only non-vanishing component of the matrix γ_{ij} is

$$\gamma_{\varphi\varphi}=\frac{b\rho_0}{\rho_0+b}. \tag{8}$$

Thus,

$$\mathrm{Tr}(\gamma)\equiv\gamma^i_i=\gamma^\varphi_\varphi=\frac{b}{\rho_0(\rho_0+b)} \tag{9}$$

and (3) becomes

$$-\mathrm{Tr}(\gamma)=8\pi S^0_0=8\pi S^3_3 \tag{10}$$

the rest of its components being vacuus.

Now, S_{00} is the surface mass-energy density σ of the shell. Therefore, from (9) and (10) we conclude that the third of the above conditions is also satisfied, provided

$$p\equiv S^3_3=-\sigma \tag{11}$$

and b is chosen such that

$$\frac{b}{\rho_0(\rho_0+b)}=8\pi\sigma. \tag{12}$$

Relation (11) is consistent with the assumption that the shell consists of straight line cosmic strings which are parallel to the axis. Equation (12), on the other hand, can be written as

$$b=\frac{8\pi\sigma\rho_0^2}{1-8\pi\sigma\rho_0}. \tag{13}$$

This makes explicit the fact that σ and ρ_0 are the only free parameters of the physical problem at hand, and that the range of these parameters is restricted by the condition

$$8\pi\sigma\rho_0 < 1. \tag{14}$$

Let us now turn to the properties of spacetime in the cylinder's exterior. According to (7), spacetime outside the cylindrical wall is flat. Equation (6), on the other hand, implies that the $\mathrm{d}t = \mathrm{d}z = 0$ hypersurfaces have an angle deficit D, where

$$D = 2\pi - \frac{2\pi\rho_0}{\rho_0 + b} = \frac{2\pi b}{\rho_0 + b}. \tag{15}$$

Using (12), this can be written as

$$D = (4\pi)^2\sigma\rho_0 \tag{16}$$

or as

$$D = 8\pi\mu \tag{17}$$

where

$$\mu \equiv 2\pi\rho_0\sigma \tag{18}$$

is the mass-energy per unit z length of the cylinder. In terms of this parameter, on the other hand, condition (14) can be expressed as

$$\mu < \tfrac{1}{4} \tag{19}$$

which is identical with the corresponding result obtained in connection with solid cylinder and straight line models of cosmic strings [1].

Finally, let us substitute (18) into (13) to obtain

$$b = h\rho_0 \qquad h \equiv 4\mu/(1 - 4\mu). \tag{20}$$

This allows us to write the line element (2) in the form

$$\mathrm{d}s_+^2 = -\mathrm{d}t^2 + \mathrm{d}\rho^2 + \left(\frac{\rho + h\rho_0}{1 + h}\right)^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2. \tag{21}$$

Letting ρ_0 and b vanish simultaneously we obtain the spacetime metric outside the straight line cosmic string stretched along the z axis [1].

References

[1] Vilenkin A 1981 *Phys. Rev.* D **23** 852
Linet P S 1987 *Gen. Rel. Grav.* **17** 1109
Gott J R III 1985 *Astrophys. J.* **288** 422
Hiscock W A 1985 *Phys. Rev.* D **31** 3288

[2] Futamase T and Garfinkle D 1988 *Phys. Rev.* D **37** 2086

[3] Letelier P S 1987 *Class. Quantum Grav.* **4** L75

[4] Xanthopoulos B C 1986 *Phys. Lett.* **178B** 163

[5] Misner C W, Thorn K S and Wheeler J A 1973 *Gravitation* (San Francisco: Freeman) § 21.13